Q 5 Differentiate the function

(a) $f(s) = \ln \ln s$

$\frac{d}{dx} \ln(u) = \frac{1}{u} \cdot u' = \frac{u'}{u}$

$u = \ln s$

$u' = \frac{1}{s} \cdot 1$

$u' = \frac{1}{s}$

$f'(s) = \frac{1}{u} \times u'$

$= \frac{1}{\ln(s)} \cdot 1/s$

$f'(s) = \frac{1}{s \ln s}$

(b) $f(\theta) = \cos \theta^2$

$\frac{d}{d\theta} \cos \theta^2 = \frac{d}{d\theta} [f(g(x))] = \frac{d}{d[g(x)]} f(x) \cdot \frac{d}{dx} [g(x)]$

$\frac{d}{d\theta} [\cos \theta] = -\sin\theta$ and by power rule

$\frac{d}{d\theta} (\theta^2) = 2\theta$

$\frac{d}{d\theta} [\cos \theta^2] = -\sin\theta^2 \cdot \frac{d}{dx} (\theta^2)$

$\frac{d}{d\theta} \cos\theta^2 = -2\theta \sin\theta^2$

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(c) $f(t) = \dfrac{1}{(2t+1)^2}$

$= f(t) = (2t+1)^{-2}$

$\dfrac{d}{dt}(2t+1)^{-2} = -2(2t+1)^{-3} \cdot 2$

$= \dfrac{-4}{(2t+1)^3}$

(d) ~~$f(t) = 2t^3$~~

$f(t) = 2^{t^3}$.

$\dfrac{d}{dt}\left(2^{t^3}\right) = \dfrac{d}{dt}\left(e^{\ln 2}\right)^{t^3}$

$= \dfrac{d}{dt}\left[e^{(\ln 2)t^3}\right.$

$= e^{(\ln 2)t^3} \cdot \ln 2$

$= e^{(\ln 2)t^3} \cdot \ln 2$

$= 2^{t^3} \cdot \ln 2$

$= 2^{t^3} \ln 2$

Question 6

Find the local maximum and minimum using both first and 2nd derivative

$f x = 1 + 3x^2 - 2x^3$.

$f'(x) = 6x - 6x^2$

$6x - 6x^2 = 0$

$6x(1-x) = 0$

$6x = 0$

$x = 0$. or $x = 1$

Points $x = 0$ and $x = 1$

$f(x) = 1 + 3x(0) + 2x(0)$

$= 1$

at point $x = 1$, $f(x) = 1 + 3 - 2$

2

Maxima at $x = 1$ and minima at $x = 2$

Using 2nd derivative

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

$6 - 12x = 0$

$x = \frac{1}{2}$

$f(x) = 1 + 3x(0.5)^2 - 2x(0.5)^3$

$1 + 0.75 + 0.25$

$= 2$

Maxima at $x = \frac{1}{2}$

(b) $f(x) = \dfrac{x^2}{x-1}$

$f' = x^2(x-1)^{-1}$

$f(x) = x^2$ and $g x = x - 1$

$$= \frac{(x-1)\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{2\cdot(x-1)\ x - x^2}{(x-1)^2}$$

$\cancel{y = \frac{x^2}{x}}$

$$\frac{2(x-1)x - x^2}{(x-1)^2} = 0$$

$2(x-1)x - x^2 = 0$

$2x^2 - 2x - x^2 = 0$

$\cancel{2x}$ $\quad x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$ or $x = 2$

at $x = 0$ we have minima

at $x = 2$, $f(x)' = \dfrac{4}{1}$

Maximum at 4 where $x = 2$